The effect of intrinsic birefringence in deep UV-lithography

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ABSTRACT

The subject of birefringence induced by spatial dispersion (BISD), also called intrinsic birefringence, recently became an important issue for 157-nm lithography. For the deep UV range, because of intrinsic absorption, only crystalline materials can be used as optical materials for lens manufacturing. The physical properties of crystals are basically affected by spatial dispersion, especially at very short wavelengths. The resulting BISD leads to a serious deterioration of optical image quality.

Recently the mathematical formalism for analyzing those aspects of the BISD effect that are relevant for optical design has been published. In this work we give an equivalent but simplified derivation of these results. This mathematical formalism is then applied to optical system design and the correction methodology is discussed. An example of optical system is given that has been corrected for the BISD effect.

Keywords: lithography, birefringence, crystal, optical system design

1. INTRODUCTION

The phenomenon of birefringence in certain types of crystals has been known for more than three centuries. It was first discovered by Bartholinus, qualitatively explained by Huygens and marvellously described quantitatively by Fresnel. A firm foundation for the phenomenon of birefringence was obtained by applying Maxwell's laws to crystalline media with specific symmetry properties. In this classical description, cubic crystals do not show birefringence and, indeed, for most purposes they can effectively be considered as being isotropic.

It was Lorentz who first indicated the presence of anisotropy introduced by spatial dispersion in cubic crystals as early as 1878 [1]. This observation was made again in [2] on the basis of a microscopic investigation of quadripolar transitions in crystals, and in [3] on the basis of macroscopic electrodynamics. The detailed analysis of this problem has been carried out in the first edition of Ref. [4]. It was shown, among others, that cubic crystals possess seven optical axes (the three main crystallographic axes and the four body diagonals of the cube). In this respect cubic crystals can be called heptaxial.

A number of experimental and theoretical studies were carried out in the 1970's on the birefringence effect induced by spatial dispersion in semiconductors (see [4], Section 4.6.2). Because of the very small size of the effect and the absence of any practical applications these investigations have not been done in much detail.

Recent publications by Burnett et al demonstrate the great practical importance of the phenomenon [5,6]. The birefringence induced by spatial dispersion has been measured and calculated for CaF_2 and BaF_2 in the ultraviolet part of the spectrum. It was shown that the magnitude of the BISD in these cubic crystals is sufficiently large to cause serious problems when using CaF_2 for precision UV optical systems at wavelengths as short as 157 nm [6]. The birefringence effect is extensively described and quantified in these references. However some aspects would profit from a further analysis. In Section 2 of this paper we present a different theoretical approach to the BISD subject which allows us to simplify the derivations and to obtain fully equivalent analytic expressions of the final formulae for cubic crystals

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(Section 3). In the final fourth Section we present our calculation method for the BISD and the compensation of its detrimental effect on the image quality in a lithographic objective.

2. BIREFRINGENCE IN CRYSTALLINE MEDIA

We will use the macroscopic electrodynamic approach [4] to analyze the birefringence in crystals. The macroscopic Maxwell equations for the electromagnetic field quantities in a medium are written as

$$\left.\begin{array}{c} \text{rot } \mathbf{B} = \dfrac{1}{c}\dfrac{\partial \mathbf{D}}{\partial t} + \dfrac{4\pi}{c}\mathbf{j}^{ext} \\[2ex] \text{div } \mathbf{D} = 4\pi\rho^{ext} \\[2ex] \text{rot } \mathbf{E} = -\dfrac{1}{c}\dfrac{\partial \mathbf{B}}{\partial t} \\[2ex] \text{div } \mathbf{B} = 0. \end{array}\right\} \tag{1}$$

Here $\mathbf{E}$ is the electric field strength, $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic induction. The quantities $\mathbf{j}^{ext}$ and ρ^{ext} are the external current and charge densities which are sources of the external electromagnetic field. These equations are supplemented by the relation between the electric induction $\mathbf{D}$ and the electric field $\mathbf{E}$. This material equation can be written in the framework of macroscopic electrodynamics in the following general form

$$D_i(\mathbf{r},t) = \int\limits_{-\infty}^{t} dt' \int d\mathbf{r}' \varepsilon_{ij}(t-t',\mathbf{r}-\mathbf{r}')E_j(\mathbf{r}',t'), \tag{2}$$

where $\varepsilon_{ij}(t,\mathbf{r})$ is the dielectric tensor of the medium.

Here we have used the principle of causality according to which the induction at time t is only determined by the present field and the field at previous times $t' \leq t$.

We Fourier transform the Eqs. (1) and (2) assuming that

$$E_i(\mathbf{r},t) = \int E_i(\omega,\mathbf{k})e^{i(\mathbf{kr}-\omega t)}\,d\omega\,d\mathbf{k} \tag{3}$$

For the other quantities we use the same notation as well. We then obtain

$$\mathbf{D}(\omega,k) = -\frac{c}{\omega}\big(\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k})\big), \tag{4}$$

$$\mathbf{k}\cdot\mathbf{D}(\omega,k) = 0, \tag{5}$$

$$\mathbf{B}(\omega,\mathbf{k}) = \frac{c}{\omega}\big(\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k})\big), \tag{6}$$

$$\mathbf{k}\cdot\mathbf{B}(\omega,k) = 0, \tag{7}$$

and

$$D_i(\omega, \mathbf{k}) = \varepsilon_{ij}(\omega, \mathbf{k}) E_j(\omega, \mathbf{k}), \tag{8}$$

where summation over the index j, appearing twice, is assumed.

We can also introduce the inverse dielectric function matrix $\varepsilon_{ij}^{-1}(\omega, \mathbf{k})$ and write

$$E_i(\omega, \mathbf{k}) = \varepsilon_{ij}^{-1}(\omega, \mathbf{k}) D_j(\omega, \mathbf{k}). \tag{9}$$

Writing Eqs. (4)-(7) we have neglected the external currents and charges because they are not important for the main goal of our investigation, namely the dependence of the dielectric function matrix on the wave vector $\mathbf{k}$ taking into account the spatial dispersion effect. The spatial dispersion is determined by the parameter $a\mathbf{k}$ or by the somewhat more descriptive parameter a/λ, where a is a characteristic dimension (the radius of "the region of influence", radius of molecular action, etc). and λ is the wavelength of the electromagnetic field. In a condensed non-metallic medium the radius a is about the order of the lattice constant. Therefore, the parameter a/λ is very small, even in the optical or ultraviolet range of the electromagnetic spectrum.

Eliminating the magnetic induction $\mathbf{B}$ from the Eqs. (4) - (7) we obtain the expression

$$\mathbf{D} = -\frac{c^2}{\omega^2}[\mathbf{k} \times (\mathbf{k} \times \mathbf{E})] = \frac{c^2}{\omega^2}[k^2\mathbf{E} - \mathbf{k} \cdot (\mathbf{k} \cdot \mathbf{E})], \tag{10}$$

and, substituting the Eq. (8) into (10), we find

$$\left[\frac{\omega^2}{c^2}\varepsilon_{ij}(\omega, \mathbf{k}) - k^2\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\right]E_j(\omega, \mathbf{k}) = 0. \tag{11}$$

If we use the matrix $\varepsilon_{ij}^{-1}(\omega, \mathbf{k})$ we obtain

$$\left[\frac{\omega^2}{c^2}\delta_{ij} - k^2\left(\delta_{il} - \frac{k_i k_l}{k^2}\right)\varepsilon_{lj}^{-1}(\omega, \mathbf{k})\right]D_j(\omega, \mathbf{k}) = 0. \tag{12}$$

These homogeneous systems of algebraic equations have nontrivial solutions $\mathbf{E}(\omega, k) \neq 0$ and $\mathbf{D}(\omega, k) \neq 0$, only if the corresponding determinants vanish

$$\left|\frac{\omega^2}{c^2}\varepsilon_{ij}(\omega, \mathbf{k}) - k^2\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\right| = 0, \tag{13}$$

and

$$\left|\frac{\omega^2}{c^2}\delta_{ij} - k^2\left(\delta_{il} - \frac{k_i k_l}{k^2}\right)\varepsilon_{lj}^{-1}(\omega, \mathbf{k})\right| = 0. \tag{14}$$

The dispersion equations (13) and (14) give the relation between ω and $\mathbf{k}$ for the electromagnetic normal waves (or eigenwaves) in a given medium for $\omega_l = \omega_l(\mathbf{k})$ when $l = 1, 2, \ldots$, where the subscript corresponds to the given normal wave. For these waves, assuming that they are normal, we can write the wave vector $\mathbf{k}$ in the form

$$\mathbf{k} = \frac{\omega}{c} n(\omega, \mathbf{s}) \mathbf{s}, \tag{15}$$

where $\mathbf{s}$ is the unit vector in the direction $\mathbf{k}$ and $n(\omega, \mathbf{s})$ is the corresponding refraction index. The dispersion equation (13) can be conveniently written in the form

$$\left| n^2(\omega,\mathbf{s})\left(\delta_{ij} - s_i s_j\right) - \varepsilon_{ij}\left(\omega, \frac{\omega}{c} n(\omega,\mathbf{s})\mathbf{s}\right)\right| = \varepsilon_{ij} s_i s_j n^4 - \left[\left(\varepsilon_{ij} s_i s_j\right)\varepsilon_{ll} - \varepsilon_{il}\varepsilon_{lj} s_i s_j\right]n^2 + \left|\varepsilon_{ij}\right| = 0. \tag{16}$$

This is the fundamental equation of crystal optics. In classical crystal optics $\varepsilon_{ij} = \varepsilon_{ij}(\omega)$ and (16) becomes quadratic with respect to n^2 and this reduced form is frequently called Fresnel's equation.

For our purpose, it is more convenient to investigate the dispersion equation (14) because we can use the property that the electric induction $\mathbf{D}$ is always transverse for normal waves. It means that we can choose the coordinate system whose z-axis is directed along $\mathbf{s}$ and then the vector $\mathbf{D}$ will have only two components D_x and D_y. In the adopted coordinate system the wave equation and the dispersion equation have the following form (taking $s_1 = s_2 = 0$, $s_3 = 1$)

$$m^2 D_\alpha = \varepsilon_{\alpha\beta}^{-1} D_\beta, \tag{17}$$

$$\left| m^2 \delta_{\alpha\beta} - \varepsilon_{\alpha\beta}^{-1}\right| = m^4 - \left(\varepsilon_{11}^{-1} + \varepsilon_{22}^{-1}\right)m^2 + \varepsilon_{11}^{-1}\varepsilon_{22}^{-1} - \left(\varepsilon_{12}^{-1}\right)^2 = 0 \tag{18}$$

in which we introduced the notation

$$\frac{1}{n^2} = m^2, \ \alpha, \beta = 1, 2. \tag{19}$$

The dispersion equation (18) has two roots for the quantity m^2

$$m_{1,2}^2 = \frac{\varepsilon_{11}^{-1} + \varepsilon_{22}^{-1}}{2} \pm \frac{1}{2}\sqrt{\left(\varepsilon_{11}^{-1} - \varepsilon_{22}^{-1}\right)^2 - 4\left(\varepsilon_{12}^{-1}\right)^2}. \tag{20}$$

and, consequently, we are led to two mutually orthogonal vectors $\mathbf{D}_1$ and $\mathbf{D}_2$. As it is well-known from [4] and [7], the existence of two values of the refractive index n for a given direction of the wave vector $\mathbf{k}$ is the origin of the appearance of the birefringence effect. The dispersion equation (18) has a multiple root $m_1^2 = m_2^2$ if

$$\left(\varepsilon_{11}^{-1} - \varepsilon_{22}^{-1}\right)^2 - 4\left(\varepsilon_{12}^{-1}\right)^2 = 0 \tag{21}$$

and the birefringence is absent in this case. Also it is well-known from [4] and [7] that in the case of classical crystal optics, i.e. for $\varepsilon_{ij} = \varepsilon_{ij}(\omega)$, the multiple root for m exists for every direction of $\mathbf{k}$ only in the case of cubic crystals. For all other crystals with a lower symmetry, birefringence is absent only for the wave orthogonal to the principal optical axis. For hexagonal, tetragonal or trigonal crystals there is one such an axis and these crystals are called uniaxial. For the

three remaining crystal systems, namely the orthorhombic, monoclinic and triclinic ones, there are two privileged normal wave directions for which there is no birefringence and the crystals are called biaxial. The situation in this respect is more complicated when the spatial dispersion is taken into account, i.e. if the dielectric function matrix depends on both the frequency ω and the wave vector $\mathbf{k}$.

3. SPATIAL DISPERSION OF CUBIC CRYSTALS

First of all we write the inverse dielectric matrix for cubic crystals in the case of small spatial dispersion in the usual form

$$\varepsilon_{ij}^{-1}(\omega,\mathbf{k}) = \varepsilon_{ij}^{-1}(\omega)\delta_{ij} + \beta_{ijlm}(\omega)k_l k_m , \tag{22}$$

or as

$$\varepsilon_{ij}^{-1}(\omega,\mathbf{k}) = \varepsilon_{ij}^{-1}(\omega)\delta_{ij} + \beta_{ijlm}(\omega)\frac{\omega^2}{c^2}n^2 s_l s_m . \tag{23}$$

The fourth-rank tensor β_{ijlm} has only three independent and non-zero components for cubic crystals with symmetry classes O, T_d and O_h [4, 8]. These are

$$\beta_1 = \beta_{xxxx} = \beta_{yyyy} = \beta_{zzzz} , \tag{24}$$

$$\beta_2 = \beta_{xxzz} = \beta_{yyxx} = \beta_{zzyy} = \beta_{zzxx} = \beta_{xxyy} = \beta_{yyzz} , \tag{25}$$

$$\beta_3 = \beta_{xyxy} = \beta_{yzyz} = \beta_{zxzx} . \tag{26}$$

Using these expressions the matrix elements of ε_{ij}^{-1} can be written as

$$\varepsilon_{xx}^{-1} = \varepsilon^{-1}(\omega) + \left(\frac{\omega}{c}n\right)^2 \left[\beta_1 s_x^2 + \beta_2\left(s_y^2 + s_z^2\right)\right] , \tag{27}$$

$$\varepsilon_{xy}^{-1} = 2\left(\frac{\omega}{c}n\right)^2 \beta_3 s_x s_y , \tag{28}$$

$$\varepsilon_{yy}^{-1} = \varepsilon^{-1}(\omega) + \left(\frac{\omega}{c}n\right)^2 \left[\beta_2\left(s_x^2 + s_z^2\right) + \beta_1 s_y^2\right] , \tag{29}$$

$$\varepsilon_{xz}^{-1} = 2\left(\frac{\omega}{c}\tilde{n}\right)^2 \beta_3 s_x s_z , \tag{30}$$

$$\varepsilon_{zz}^{-1} = \varepsilon^{-1}(\omega) + \left(\frac{\omega}{c}n\right)^2 \left[\beta_2\left(s_x^2 + s_y^2\right) + \beta_1 s_z^2\right] , \tag{31}$$

$$\varepsilon_{yz}^{-1} = 2\left(\frac{\omega}{c}n\right)^2 \beta_3 s_y s_z. \tag{32}$$

The factor two in the expressions for ε_{xy}^{-1}, ε_{xz}^{-1} and ε_{yz}^{-1} is due to the summation in (22). The equations (22) – (32) allow us to write the expression for the inverse dielectric matrix in the form

$$\varepsilon_{ij}^{-1}(\omega,\frac{\omega}{c}n\mathbf{s}) = \left(\varepsilon^{-1}(\omega) + \frac{\omega^2}{c^2}n^2\beta_2\right)\delta_{ij} + \frac{\omega^2}{c^2}n^2 \tilde{\beta} s_i^2 \delta_{ij} + 2\frac{\omega^2}{c^2}n^2\beta_3 s_i s_j, \tag{33}$$

where

$$\tilde{\beta} = \beta_1 - \beta_2 - 2\beta_3. \tag{34}$$

The first term in the expression (33) is the isotropic contribution, the second one is anisotropic, but it is expressed in terms of a diagonal matrix; the last term is purely longitudinal. If we rewrite the wave equation (14)

$$\left[\frac{\delta_{ij}}{n^2(\omega,\mathbf{s})} - \left(\delta_{il} - s_i s_l\right)\varepsilon_{lj}^{-1}(\omega,\frac{\omega}{c}n\mathbf{s})\right]D_j(\omega,\frac{\omega}{c}n\mathbf{s}) = 0, \tag{35}$$

it follows after some simple algebra that the longitudinal part of ε_{ij}^{-1} will disappear from this equation due to the prefactor $\left(\delta_{il} - s_i s_l\right)$, because the multiplication of this prefactor with the longitudinal part yields zero.

We will solve Eq. (35) with the aid of perturbation theory and rewrite this equation as

$$\left(L_0 + L_1\right)_{ij} D_j = \rho(\omega)\delta_{ij} D_j \tag{36}$$

and

$$\rho(\omega) = \rho_0(\omega,\mathbf{s}) + \rho_1(\omega,\mathbf{s}), \tag{37}$$

where

$$\rho(\omega) = \frac{1}{n^2(\omega,\mathbf{s})} \tag{38}$$

is the eigenvalue of this equation which should be calculated up to the first order perturbations described by the perturbation operator L_1. It has the form

$$\left(L_1\right)_{ij} = \frac{\omega^2}{c^2}n^2(\omega,\mathbf{s})\tilde{\beta} s_i^2 \delta_{ij} - \frac{\omega^2}{c^2}n^2(\omega,\mathbf{s})\tilde{\beta} s_i s_j. \tag{39}$$

In the framework of perturbation theory, we can change the value $n^2(\omega,\mathbf{s})$ in Eq. (39) into $n_0^2(\omega)$ defined by the zero-order approximation. This zero-order perturbation is defined by the equation

$$\left(L_0\right)_{ij} D_j^0 = \rho_0(\omega)D_i^0, \text{ where} \tag{40}$$

$$\left(L_0\right)_{ij} = \left(\varepsilon^{-1}(\omega) + \frac{\omega^2}{c^2} n_0^2 \beta_2\right)\left(\delta_{ij} - s_i s_j\right) \tag{41}$$

and, consequently, Eq. (40) for the zero-order approximation can be written as

$$\left[\left(\varepsilon^{-1}(\omega) + \frac{\omega^2}{c^2} n_0^2 \beta_2\right)\left(\delta_{ij} - s_i s_j\right)\right] D_j^0 = \rho_0(\omega) D_i^0. \tag{42}$$

By using Eq. (5) this equation reduces to the wave equation for isotropic media, which has the multiple root for $\rho_0(\omega)$ equal to

$$\rho_0(\omega) = \frac{1}{n_0^2(\omega)} = \varepsilon^{-1}(\omega) + \frac{\omega^2}{c^2} n_0^2(\omega)\beta_2. \tag{43}$$

Moreover, in the framework of macroscopic electrodynamics we can neglect the second term in the right part of (43) because there are no possible experiments which can help us to distinguish the terms $\varepsilon^{-1}(\omega)$ and $\frac{\omega^2}{c^2} n_0^2(\omega)\beta_2$. It means that for the refraction index $n_0(\omega)$ in the zero-order approximation we can write

$$n_0^2(\omega) = \varepsilon(\omega), \tag{44}$$

where

$$\varepsilon(\omega) = \left(\varepsilon^{-1}(\omega)\right)^{-1}. \tag{45}$$

The existence of the multiple root of $\rho_0(\omega)$ in the zero-order approximation tells us that the system of equations (36) and (37) has a degenerate kernel [9] and we should use for the calculation of the first order correction to $\rho_1(\omega)$ a so-called secular equation. This equation will give also the two correct values of $\tilde{D}_i{}^0(\omega, n(\omega, \mathbf{s}))$ resulting from the removing of the degeneracy due to the perturbation operator L_1. If we write the first order correction $\rho_1(\omega)$ as

$$\rho_1(\omega) = \frac{\omega^2}{c^2} n_0^2(\omega)\, \tilde{\beta}\, \tilde{\rho}_1(\omega, \mathbf{s}), \tag{46}$$

the secular equation will have the form

$$\left(s_i^2 \delta_{ij} - s_i s_j\right) e_j(\mathbf{s}) = \tilde{\rho}(\omega) e_i(\mathbf{s}), \tag{47}$$

where we introduced the unit vector $\mathbf{e}$ in the direction of $\overset{\sim}{\mathbf{D}}{}^0(\mathbf{s})$. Moreover, because the vector $\overset{\sim}{\mathbf{D}}{}^0(\mathbf{s})$ is orthogonal to the vectors $\mathbf{s}$, Eq. (47) can be simplified to

$$s_i^2 e_i(\mathbf{s}) = \tilde{\rho}_1(\omega, \mathbf{s}) e_i(\mathbf{s}). \tag{48}$$

The two solutions $\tilde{\rho}_1^{(1,2)}$ turn out to be the roots of the quadratic equation

$$\tilde{\rho}_1^2(\mathbf{s}) - 2(s_x^2 s_y^2 + s_x^2 s_z^2 + s_y^2 s_z^2)\tilde{\rho}_1(\mathbf{s}) + 3s_x^2 s_y^2 s_z^2 = 0, \tag{49}$$

More details about this step of the derivation will be given in a subsequent paper. Basically, the pair of eigenmodes $(e_1(\mathbf{s}), e_2(\mathbf{s}))$ has been chosen in such a way that in this basis the 2×2 matrix associated to the operator L_1 becomes diagonal. The two solutions for $\tilde{\rho}_1(\omega, \mathbf{s})$ i.e. the diagonal element of the matrix mentioned above are

$$\tilde{\rho}_1^{(1,2)}(\mathbf{s}) = (s_x^2 s_y^2 + s_x^2 s_z^2 + s_y^2 s_z^2) \pm \sqrt{(s_x^2 s_y^2 + s_x^2 s_z^2 + s_y^2 s_z^2)^2 - 3s_x^2 s_y^2 s_z^2}. \tag{50}$$

It is obvious that the equation for $\tilde{\rho}_1(\omega, \mathbf{s})$ has only one solution for the mentioned above seven directions of the propagation vector $\mathbf{s}$, that are the three main crystallographic axes and the four body diagonals of the cube.

We can also obtain the expression for the eigenmodes of Eq. (48). The components of the eigenmode $e_2(\mathbf{s})$, which correspond to the eigenvalue $\tilde{\rho}_1^{(2)}(\mathbf{s})$ have the following form

$$e_{2z}(\mathbf{s}) = \frac{1}{\sqrt{2}} \sqrt{s_x^2 + s_y^2 + \frac{s_x^2 s_y^2 (s_x^2 + s_y^2) - (s_x^4 + s_y^4)s_z^2}{r}}, \tag{51}$$

$$e_{2x}(\mathbf{s}) = e_{2z}(\mathbf{s}) \frac{s_y^2(s_z^2 - s_x^2) + r}{s_x s_z (s_x^2 - s_y^2)}, \tag{52}$$

$$e_{2y}(\mathbf{s}) = e_{2z}(\mathbf{s}) \frac{s_x^2(s_z^2 - s_y^2) + r}{s_y s_z (s_y^2 - s_x^2)}, \tag{53}$$

where r is

$$r = \sqrt{(s_x^2 s_y^2 + s_x^2 s_z^2 + s_y^2 s_z^2)^2 - 3s_x^2 s_y^2 s_z^2}. \tag{54}$$

For certain directions $\mathbf{s}$ zero denominators appear and then these expressions cannot be used directly. However analytical expressions can be derived for these special cases. For instance for $s_x > 0$ and $s_y > 0$, but s_x very small we obtain

$$e_{2x}(\mathbf{s}) = -\sqrt{\frac{1}{s_z^2}} s_z, \quad e_{2y}(\mathbf{s}) = \frac{s_x \sqrt{\frac{1}{s_z^2}} s_z}{2\sqrt{1 - s_z^2}}, \quad e_{2z}(\mathbf{s}) = \frac{1}{2} s_x \sqrt{\frac{1}{s_z^2}}. \tag{55}$$

The components of the eigenmode $e_1(\mathbf{s})$ corresponding to the eigenvalue $\overset{\sim}{\rho}_1{}^{(1)}(\mathbf{s})$ can be obtained from the equation

$$e_1(\mathbf{s}) = e_2(\mathbf{s}) \times \mathbf{s} . \qquad (56)$$

Our first order correction to the eigenvalue $\overset{\sim}{\rho}_1(\omega, \mathbf{s})$ coincides with that obtained by Burnett et al in [6]. We have also compared our results produced by Eqs. (51)-(54) and the ones obtained with the Fortran code by Burnett et al from [10] and, in all cases, we have obtained identical results.

If we define the variation of the birefringence with propagation direction according to Eq. (22) in [6] as

$$\Delta \overset{\sim}{\rho}_1(\mathbf{s}) = \overset{\sim}{\rho}_1{}^{(1)}(\mathbf{s}) - \overset{\sim}{\rho}_1{}^{(2)}(\mathbf{s}) = 2\sqrt{(s_x^2 s_y^2 + s_x^2 s_z^2 + s_y^2 s_z^2)^2 - 3 s_x^2 s_y^2 s_z^2} , \qquad (57)$$

this value will have the same sign for every direction. The real origin for a possible change of the sign of the birefringence variation shown in Fig. 6 of [6] is related to the behavior of the eigenmodes $e_{1,2}(\mathbf{s})$. This fact can be easily understood using the picture shown in Fig. 1.
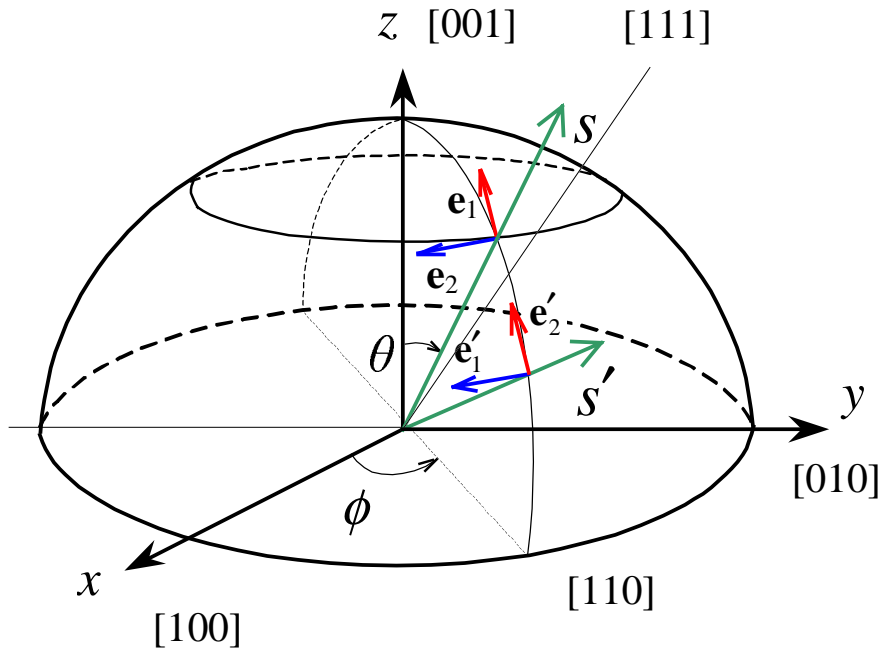

Fig. 1. The behavior of the eigenvectors in the $[\bar{1}10]$-diagonal plane of the elementary cubic cell.

Here we show the behavior of the eigenmodes $e_{1,2}(\mathbf{s})$ for two propagation directions in the diagonal plane $[\bar{1}10]$ of the cubic cell containing the [001], [111] and [110] directions. We can see that the eigenmode $e_1(\mathbf{s})$ corresponding to the larger value of $\overset{\sim}{\rho}_1(\omega, \mathbf{s})$ lies in the plane $[\bar{1}10]$ for the directions above the axis [111]. The eigenmode $e_2(\mathbf{s})$ is normal to this plane. The situation is reverse for the directions below the axis [111]. Here the eigenmode $e_2'(\mathbf{s}')$ lies in the plane $[\bar{1}10]$. It means that if we consider the difference of the refraction indices between one wave polarized in the

[110] plane and other normal to this plane this difference will have opposite sign for directions above and below axis [111]. This difference has certainly the opposite sign for the vice versa chosen waves as it was shown in Fig. 6 in [6]. Which sign, positive or negative, should be considered as physically correct is not defined. The solution of this dichotomy can be found only considering the process of the refraction of the incoming light with a given polarization on the crystal surface.

4. BISD COMPENSATION IN DUV LITHOGRAPHY

In optical system design, BISD leads to the appearance of multiple polarized rays during refraction. There are two basic effects depending on wavelength, thickness and lens shape, which follow from this fact. The first one is the angular difference in ray paths, but due to the minute value of the quantity Δn we can neglect it. The second consequence of BISD is the optical pathlength difference between rays of each image bundle (orthogonally polarized). This pathlength difference, resulting in a phase difference, can be visualized in the pupil maps, which are shown in Table 1 for the one-surface case (the dark tone means a small phase difference).
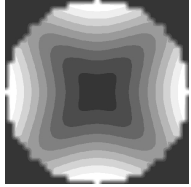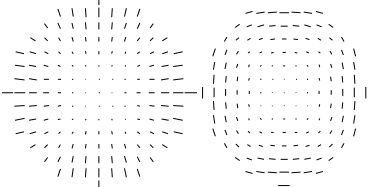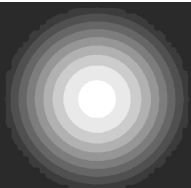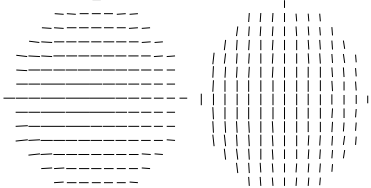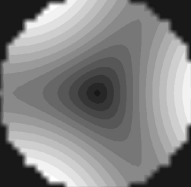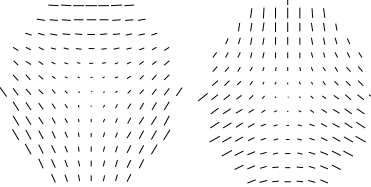
Chosen optical axis	The BISD dependence over the pupil	The projections of eigenvector states on the pupil
{100}		
{110}		
{111}		

Table 1. Directional dependence of the BISD

The first issue in optical system design with BISD is the calculation of the effect itself. Only one commercial optical design program (Code V, © Optical Research Associates) supports the calculation of the effect within a given approximation and allows to analyse the image quality taking into account the BISD. However this application has limitations, e.g. the arbitrary choice of the crystal orientation during optimization is not supported.

The second issue that is also very important is the speed of calculations. For the adequate estimation of the BISD it is necessary to calculate through the system not less than 50 rays taking into account their polarization properties (Table 2). For a lithographic lens with about 50 surfaces it drastically increases the required time for image quality estimation.

Another issue is the search of a relevant characteristic measure of the image quality for BISD in DUV systems. It is very suitable for optical system design to define the influence of BISD on the image quality by one number. In the framework

of this research it was found that a very good description for the influence of BISD is the standard deviation of the retardation (phase difference) over the pupil. This is confirmed by calculation of BISD in the optical system with and without BISD compensation (Table 3). It is obvious that not only the value of Strehl Ratio but also the standard deviation of the retardation indicates adequately the loss of image quality caused by BISD.

Low BISD	Number of rays	5	13	24	49	68	89	481	973
	Mean retardation	154.32	149.68	151.17	148.95	150.63	151.54	151.04	151.11
	standard deviation of retardation	11.174	10.717	12.075	12.235	13.449	13.546	14.136	14.103
High BISD	Number of rays	5	13	24	49	68	89	481	973
	Mean retardation	28.176	86.253	76.489	88.437	84.640	82.043	85.119	84.729
	standard deviation of retardation	35.734	59.455	41.456	49.023	49.291	51.595	49.944	49.791

Table 2. Accuracy of BISD calculation with various numbers of rays

The main obstacle for BISD compensation is the effect of asymmetry with regard to the optical axis. For preserving the optical system symmetry only the three directions {100}, {110} and {111} can be selected as an optical axis. The directions {100} and {110} have an advantage as compare as {111} because along them the BISD effect changes less when ray parameters change. However the direction {111} is more preferable from the point of view of crystal manufacturing because for this direction the residual stresses are minimal. Generally it is possible to select an arbitrary direction {abc} as an optical axis but in that case the effect calculation and the technological issues can be unpredictably difficult.

Field #	Strehl Ratio (in the absence of **BISD**)	**BISD** uncompensated			**BISD** compensation		
		Strehl Ratio	mean retardation	standard deviation of retardation	Strehl Ratio	mean retardation	standard deviation of retardation
1	0.7998	**0.4498**	77.2561	**48.1635**	**0.7769**	166.787	**12.0636**
2	0.7686	**0.4242**	74.4141	**46.6192**	**0.7526**	162.652	**12.5307**
3	0.7734	**0.4328**	66.6632	**42.2149**	**0.7545**	155.734	**13.7360**
4	0.7885	**0.4550**	58.6569	**37.0975**	**0.7687**	152.190	**14.6311**
5	0.7625	**0.4439**	54.8881	**34.4967**	**0.7651**	153.498	**13.0165**

Table 3. Image quality in an optical system with and without BISD compensation

The general strategy of the effect compensation is the following. It can be observed from Table 1 that, being chosen as an optical axis, the direction {100} has the 90°- angular symmetry and {111} the 120°- symmetry. So it is possible to tune the separate components to form the minimal retardation value. However, there is a very important nuance; when we use a {111} crystal orientation we should avoid the change of sign of the angle between the incident ray and the optical axis because in this case the orientation {111} - 0° becomes {111} - rotated by 60° and vice versa. The minimum number of components for this method is four ({100} - 0°, {100} - 45°, {111} - 0°, {111} - 60°). The full effect compensation for the off-axis point is still not clear, as far as the effect behavior is difficult to predict, but in general this behavior is comparable to that of the axial point.

So the traditional approach implies using the combination of optical components with the optical axes along the crystal orientations {001} and {111}, which can be oriented at different angles with respect to the optical axis. Some combinations of these components result in a more regular phase retardation distribution, which can be adjusted or nulled out with the available optimization tools.
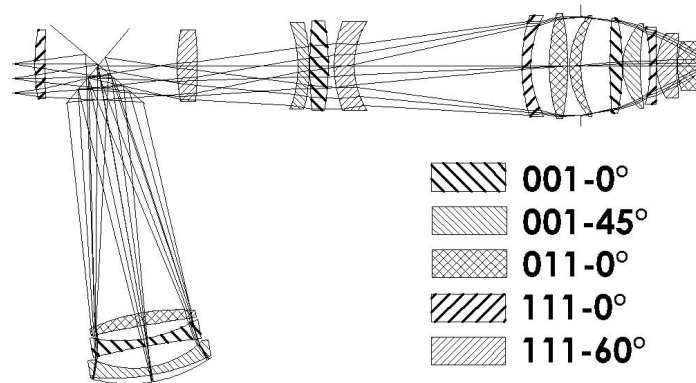

Fig 2. BISD compensation in DUV lithographic system

The novelty of our approach is the addition to this combination of the components with crystal orientation <011>. Such an arrangement allows to compensate the effect without or with minor additional optimization and does not break the aberration correction of an already geometrically optimized lens. The optical system designed according to this approach is shown in Fig 2. The image quality characteristics of this system are produced in the Table 3.

5. CONCLUSIONS

A simplified mathematical description of BISD has been given in a form that is suitable for optical design. The image quality characteristics and the correction approach have been discussed and an example of an optical system with BISD correction has been presented.

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